OCR
Oxford Cambridge and RSA

Wednesday 24 May 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is moving, with negligible air resistance, in a complete vertical circle with centre O. When P is at its highest point the speed of P is V. The horizontal line CD lies in the plane of the motion and passes through the lowest point of the circular path of P. Fig. 1 shows the particle at a point where OP makes an angle θ with the upward vertical.

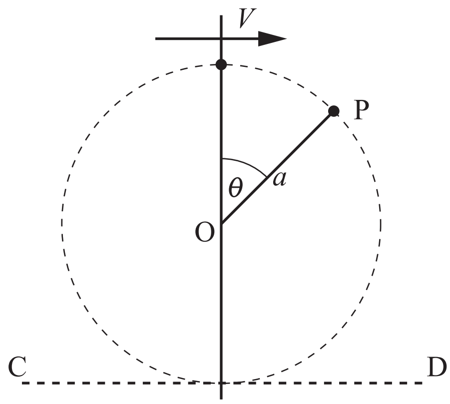

Fig. 1

(i) Show that the least possible value of V is $\sqrt{ag}$. **[2]**

(ii) Given that $V = \sqrt{ag}$, find an expression, in terms of m, g and θ, for the tension in the string when P is in the position shown in Fig. 1. **[6]**

Now consider the case $V = \sqrt{3ag}$.

(iii) Find the vertical height of P above CD when the tension in the string is equal to twice its minimum value. **[6]**

Suppose now that $V = \sqrt{kag}$, where k is a positive constant.

The string breaks if the tension in it exceeds $12mg$.

(iv) Find the set of values that k can take so that P is able to complete vertical circles. **[3]**

2 **(a)** A moving car experiences a force F due to air resistance. It is known that F depends on a product of powers of its velocity v, its cross-sectional area A and the air density ρ, and is given by

$$F = \frac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma},$$

where C is a dimensionless constant known as the drag coefficient.

(i) Write down the dimensions of force and density. **[2]**

(ii) Use dimensional analysis to find α, β and γ. **[5]**

(b)

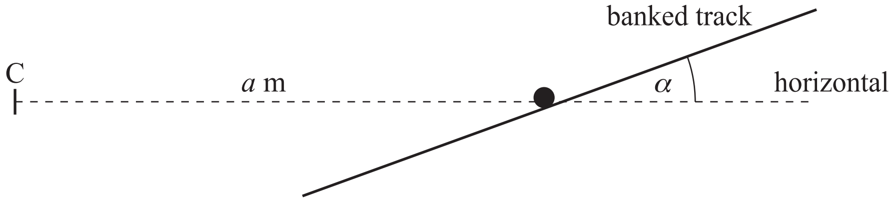

Fig. 2

A motorcyclist is riding his motorcycle around a circular banked track. The track is banked at an angle α to the horizontal, where $\tan\alpha = \frac{1}{4}$. The combined mass of the motorcycle and rider is M kg. The motion of the motorcycle and rider is modelled as a particle travelling at constant speed in a horizontal circle, with centre C and radius a m, on the banked track, as shown in Fig. 2.

(i) Given that there is no tendency for the motorcyclist to slip up or down the slope when his speed is $5\sqrt{g}\ \mathrm{m\,s^{-1}}$, show that $a = 100$. **[4]**

Suppose now that the coefficient of friction between the motorcyclist and the track is μ.

(ii) Given that the maximum constant speed for which motion in the horizontal circle centre C is possible is $28\ \mathrm{m\,s^{-1}}$, find the value of μ. **[7]**

3 Fig. 3 shows a smooth plane inclined at an angle of 30° to the horizontal. A particle P of mass 3 kg lies on the plane. One end of a light elastic string, of natural length 2 m, is attached to P and the other end is fixed to a point A. One end of a second light elastic string, of natural length 1 m, is attached to P and the other end is fixed to a point B. Both strings are made from material with modulus of elasticity 12.25 N. APB is parallel to the plane on a line of greatest slope, and the distance AB is 6 m.

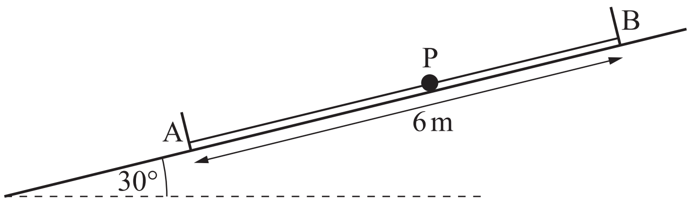

Fig. 3

The particle P moves along part of the line AB with both strings taut throughout the motion.

(i) Show that, when the extension of the string AP is x m, the tension in the string BP is $12.25(3-x)$ N. Show also that the value of x for which the system is in equilibrium is 1.2. **[4]**

The particle P is released from rest when AP = 3.35 m. At time t s, the displacement of P from its equilibrium position is y m, measured in the direction AB.

(ii) Show that the motion of P is simple harmonic with equation

$$\frac{d^2y}{dt^2} = -6.125y.$$

State the period of the motion. **[8]**

The point C is on the line AB, between A and B, such that AC = 3.1 m.

(iii) Find the speed of P when it is at C. **[2]**

(iv) Find the time elapsed after its release from rest until P is at C moving **up** the plane for the first time. **[5]**

4 Fig. 4.1 shows the shaded region R bounded by the curve $y = 2x^{-\frac{1}{2}}$ for $1 \leqslant x \leqslant 4$, the x-axis and the lines $x = 1$ and $x = 4$.

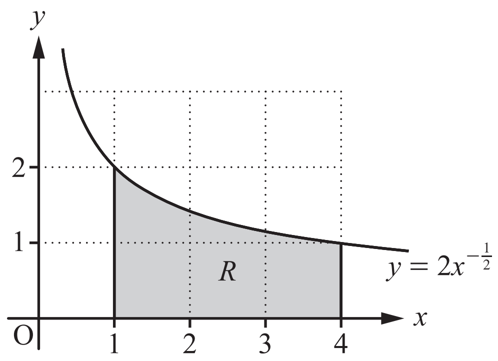

Fig. 4.1

(i) Find the exact coordinates of the centre of mass of a uniform lamina occupying the region R. **[6]**

Fig. 4.2 shows the shaded region S bounded by the curve $y = 2x^{-\frac{1}{2}}$ for $1 \leqslant x \leqslant 4$, the x-axis and the lines $x = 4$ and $y = 2x$. The line $y = 2x$ meets the curve $y = 2x^{-\frac{1}{2}}$ at the point A with coordinates (1, 2).

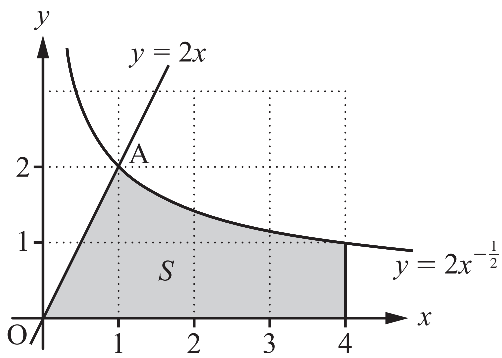

Fig. 4.2

The region S is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(ii) Show that the x-coordinate of the centre of mass of this solid is $\dfrac{39}{4(1+6\ln 2)}$.

(You may assume the standard results for the volume and the position of the centre of mass of a uniform solid cone.) **[8]**

(iii) The solid is suspended from a point on the circle described by A when S is rotated about the x-axis. Find the angle between AO and the vertical. **[4]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	
1 (ii)	

1 (iii)

1 (iv)

2 (a) (i)	
2 (a) (ii)	

2 (b) (i)

2 (b) (ii)

(answer space continued on next page)

2 (b) (ii)	**(continued)**
3 (i)	

3 (ii)	

3 (iii)	
3 (iv)	
	(answer space continued on next page)

3 (iv)	(continued)

4 (i)

4 (ii)

(answer space continued on next page)

4 (ii)	**(continued)**

4 (iii)

(answer space continued on next page)

4 (iii)	**(continued)**

OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4763**: Mechanics 3 (M3)

Advanced GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

1.	(i)		Tension (T) at highest point = 0 $(T+)mg = mV^2/a$: $V^2 = ag$	B1 E1 **[2]**	May be implied *Without further explanation,* $g = V^2/a$ *is B0E0* *But* $mg = mV^2/a$ *can earn B1E1*	
	(ii)		$T + mg\cos\theta = \frac{mv^2}{a}$ $\frac{1}{2}mv^2 - \frac{1}{2}mV^2 = mga(1-\cos\theta)$ $T = 3mg(1-\cos\theta)$.	M1 A1 M1 A1 M1 A1 **[6]**	N2L at P *Allow sign errors, sin/cos interchanged* *Condone use of V for M1A1* Energy equation [$v^2 = ag(3-2\cos\theta)$] Eliminate v^2 and use $V = \sqrt{ag}$ *Dependent on M1M1*	
	(iii)		$\frac{1}{2}mv^2 - \frac{1}{2}m(3ag) = mga(1-\cos\theta)$ Minimum T at highest point: $T = 2mg$ Using $T = 2\times 2mg$ in $T + mg\cos\theta = \frac{mv^2}{a}$ $4mg + mg\cos\theta = mg(5-2\cos\theta)$ $\cos\theta = \frac{1}{3}$ Height = $a + a\cos\theta = \frac{4a}{3}$	M1 B1 M1 A1 A1 F1 **[6]**	[$v^2 = ag(5-2\cos\theta)$] Or $\frac{1}{2}mag(4+\cos\theta) - \frac{1}{2}m(3ag) = mga(1-\cos\theta)$ *Dependent on M1M1; provided* $\cos\theta \neq 0, \pm 1$	
	(iv)		Maximum tension, at lowest point, is $12mg$ $T - mg = \frac{mv^2}{a}$ and $\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + 2mga$ $V^2 \le 7ag$: $k \le 7$ So $1 \le k \le 7$	B1 M1 A1 **[3]**	Seen or implied [$v^2 = 11ag$, $v^2 = (k+4)ag$]	

2.	**(a)**	**(i)**	Dimensions of force: MLT^{-2} Dimensions of density: ML^{-3}	B1 B1 **[2]**		
		(ii)	$MLT^{-2} = (ML^{-3})^{\alpha} (LT^{-1})^{\beta} (L^2)^{\gamma}$ Compare powers for at least one dimension $1 = \alpha$ $1 = -3\alpha + \beta + 2\gamma$ $-2 = -\beta$ $\alpha = 1, \beta = 2, \gamma = 1$	M1 A1 ft M1 A1 cao A1 cao **[5]**	All parts present, dimensions of at least v or A correct At least two equations correct All correct	
	(b)	**(i)**	Let R = normal reaction. For vertical equilibrium, $R\cos\alpha = Mg$ Horizontally, $R\sin\alpha = \frac{Mv^2}{a}$ $[= \frac{25Mg}{a}]$ Divide: $\tan\alpha = \frac{25g}{ag}$ $a = 25/\tan\alpha = 100$ AG	B1 B1 M1 E1 **[4]**	Or B2 for $mg\sin\alpha = m\frac{v^2}{a}\cos\alpha$ Or B2 for a correct triangle showing mg, R and mv^2/a *Dependent on B2*	
		(ii)	Let F = frictional force Vertically: $R\cos\alpha = Mg + F\sin\alpha$ Horizontally: $R\sin\alpha + F\cos\alpha = \frac{Mv^2}{a}$ $F = \mu R$ Eliminate R and F: $\frac{v^2}{ag} = \frac{\mu + \tan\alpha}{1 - \mu\tan\alpha}$ Solving to obtain μ $\mu = \frac{11}{24}$ (= 0.458(3))	M1 M1 A1 M1 M1 M1 A1 cao **[7]**	All terms present, allow F in wrong direction, sin/cos interchange All terms present, allow F in wrong direction, sin/cos interchange Both resolutions correct OR: solve for F and R *This M1 is dependent on first two M1M1* [$F = \frac{Mv^2}{a}\cos\alpha - Mg\sin\alpha$, $R = \frac{Mv^2}{a}\sin\alpha + Mg\cos\alpha$] OR: use $\mu = \frac{F}{R} = \frac{7.84 - 9.8\tan\alpha}{7.84\tan\alpha + 9.8}$ *Dependent on M4*	

3.	(i)		Length of BP = $4-x$: extension $3-x$ $T_{BP}=\frac{12.25(3-x)}{1}=12.25(3-x)$ AG In equilibrium: $T_{BP}=T_{AP}+3g\sin 30^\circ$ $12.25(3-x)=\frac{12.25}{2}x+\frac{3g}{2}$ $x=1.2$ AG	E1 M1 A1 E1 **[4]**	 Solve (or evaluate terms)	
	(ii)		Tension in BP = $12.25(1.8-y)$ Tension in AP = $6.125(1.2+y)$ $T_{BP}-T_{AP}-mg\sin 30^\circ=m\ddot{y}$ $12.25(1.8-y)-6.125(1.2+y)-\frac{3g}{2}=3\ddot{y}$ $\ddot{y}=-6.125y$ Period = $\frac{2\pi}{\sqrt{6.125}}=2.53879....$ (s)	M1 A1 A1 M1 A1 A1 cao E1 B1 **[8]**	Hooke's law for one string Use N2L with at least 3 terms present Allow $-m\ddot{y}$ or ma on RHS Allow $\frac{4\sqrt{2}}{7}\pi$ *Working with x earns max M0M1A1A0E0B1 unless* $x=1.2+y$ *is used at some stage*	
	(iii)		$v^2=6.125\left(0.15^2-0.1^2\right)$ $v^2=0.07656$: $v=0.276699.....$	M1 A1 **[2]**	Using $v^2=\omega^2(a^2-y^2)$ or $y=a\cos\omega t$ and $v=-a\omega\sin\omega t$ Or energy equation with all terms present	
	(iv)		$y=a\cos\omega t$ $-0.1=0.15\cos\sqrt{6.125}t$ $t=0.929(55)$ When P is moving *up* the plane, time = period − t 1.60924...... (s)	M1 A1 ft A1 M1 A1 cao **[5]**	For $y=a\cos\omega t$, $y=a\sin\omega t$, $v=(-)a\omega\sin\omega t$ or $v=a\omega\cos\omega t$ Allow ±0.1 and sin or cos; o.e. for $v=\pm 0.277$ Any solution; e.g. 0.295, 0.340, 1.564, 1.61, 2.20 seen o.e. e.g. 3*T*/4 – 0.295, *T*/2 + 0.340, 5*T*/4 – 1.564, 3*T*/2 – 2.20	

4.	(i)		$A\bar{x} = \int_1^4 x.2x^{-0.5}\mathrm{d}x = \left[\frac{4}{3}x^{1.5}\right]_1^4$	M1	For $\int x\left(2x^{-\frac{1}{2}}\right)\mathrm{d}x$	
			$= \frac{28}{3}$	A1		
			$A = \int_1^4 2x^{-0.5}\mathrm{d}x = \left[4x^{0.5}\right]_1^4 = 4$	B1		
			$\bar{x} = \frac{7}{3}$	A1		
			$A\bar{y} = \frac{1}{2}\int_1^4 \left(2x^{-0.5}\right)^2 \mathrm{d}x = \left[2\ln x\right]_1^4 = 2\ln 4$	M1	For $\int \frac{1}{2}\left(2x^{-\frac{1}{2}}\right)^2 \mathrm{d}x$	
			$\bar{y} = 2\ln 4 \div 4 = \ln 2$	A1 **[6]**	Accept $\frac{1}{2}\ln 4$	
	(ii)		Consider solid formed by rotating R:			
			$V = \pi\int_1^4 y^2\mathrm{d}x = 8\pi\ln 2$	B1	Ft result obtained in **(i)** $\times 2\pi$	
			$V\bar{x} = \pi\int_1^4 x.4x^{-1}\mathrm{d}x$	M1	For $\int x\left(2x^{-\frac{1}{2}}\right)^2 \mathrm{d}x$	
			$= \pi\left[4x\right]_1^4$	M1	For $\left[4x\right]$	
			$= 12\pi$	A1	For *12π Implied by correct $\bar{x}$*	
			$\bar{x} = \frac{3}{2\ln 2}$			
			Consider solid formed by rotating line $y = 2x$ as cone.			
			Volume = $\frac{4\pi}{3}$: com $\frac{3}{4}$ from y-axis	B1		

			Answer	Marks	Guidance	
			<table><tr><td></td><td>*Cone*</td><td>*Solid from R*</td><td>*Whole*</td></tr><tr><td>Volume</td><td>$\frac{4}{3}\pi$</td><td>$8\pi\ln 2$</td><td>$\frac{4}{3}\pi + 8\pi\ln 2$</td></tr><tr><td>Dist. of com from vertex</td><td>$\frac{3}{4}$</td><td>$\frac{3}{2\ln 2}$</td><td>$\bar{X}$</td></tr></table>			
			$\bar{X}\left(\frac{4\pi}{3} + 8\pi\ln 2\right) = \frac{4}{3}\pi \times \frac{3}{4} + 8\pi\ln 2 \times \frac{3}{2\ln 2}$	M1 A1 ft	Formula for com of composite body	
			$\bar{X} = \frac{39}{4(1+6\ln 2)}$ AG	E1 **[8]**		
		OR	Consider as a single object $V = \pi\int_0^1 (2x)^2 \mathrm{d}x + \pi\int_1^4 (2x^{-\frac{1}{2}})^2 \mathrm{d}x = \frac{4}{3}\pi + 8\pi\ln 2$ $V\bar{X} = \pi\int_0^1 x(2x)^2 \mathrm{d}x + \pi\int_1^4 x(2x^{-\frac{1}{2}})^2 \mathrm{d}x = \pi + 12\pi$ $\bar{X} = \frac{13\pi}{\frac{4}{3}\pi + 8\pi\ln 2}$ $\bar{X} = \frac{39}{4(1+6\ln 2)}$ AG		B1 ft for $8\pi\ln 2$ M1M1A1 for 12π (as above) B1 for $\frac{4}{3}\pi$ and π M1A1 ft E1	
	(iii)		Find angle between AG and line $x = 1$ $\tan\theta = \frac{\bar{X} - 1}{2}$	M1	Award M2A2 for other methods (e.g. triangle OAG)	
			$\theta = 23.987724...^\circ$	A1		
			Identify required angle as $\theta + \tan^{-1}\frac{1}{2}$	M1		
			Angle between AO and vertical = $50.552775...^\circ$	A1 cao **[4]**	Accept 0.882 rad	

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4763 Mechanics 3

General Comments:

The marks on this paper were generally high, with an average mark of about 75%, and a fair number of candidates scored full marks. The majority of candidates appeared to have sufficient time to complete the paper, and were able to demonstrate a good understanding of most of the topics being examined. Questions 2 (on dimensional analysis and motion in a horizontal circle) and 4 (on centres of mass) were answered slightly better than questions 1 (on motion in a vertical circle) and 3 (on simple harmonic motion).

Comments on Individual Questions:

Q.1(i) The least possible speed was almost always derived correctly. As the answer was given it was of course necessary to give an adequate explanation, and some candidates failed to do this.

Q.1(ii) This was answered well, with the conservation of energy and the radial equation of motion being used to obtain the tension in terms of θ. There were sometimes errors in the potential energy term, and in the algebra. However, the most common error was to ignore energy altogether and assume that the motion was at constant speed $\sqrt{(ag)}$.

Q.1(iii) Most candidates found the minimum tension (at the top of the circle) correctly. It was then necessary to use the radial equation of motion and the conservation of energy, in a similar way to part (ii), to obtain the value of θ and hence the vertical height. Many candidates managed this successfully, although some assumed that the speed was constant. Another serious error was to use the formula for the tension obtained in part (ii), not realising that it was no longer valid as the value of V had changed.

Q.1(iv) Most candidates used the fact that the maximum tension occurred at the lowest point. Again several assumed that the speed was constant, but the majority used conservation of energy to obtain the upper limit $k \le 7$. However, the lower limit $1 \le k$, which follows from part (i), was usually omitted.

Q.2(a)(i) Almost every candidate gave the dimensions of force and density correctly.

Q.2(a)(ii) The method for finding powers in a formula by dimensional analysis was very well understood, and usually carried out accurately. There were a few careless arithmetic and algebraic errors. Some candidates took the dimensions of velocity to be MT^{-1}, presumably because the units of length are metres (m).

Q.2(b)(i) This part was quite well answered. The radial equation was usually written correctly; however, many resolved perpendicular to the track ($R = mg\cos\alpha$) instead of vertically ($R\cos\alpha = mg$). Because of the given answer, a full explanation was expected, and quoting a formula such as $\tan\alpha = v^2/(ag)$ did not earn any marks.

Q.2(b)(ii) Here it was necessary to consider friction (F) and the normal reaction (R), to resolve vertically and use the radial equation; and then use $F = \mu R$. Many candidates did this neatly and efficiently, and some eventually arrived at the correct answer after a very lengthy and complicated piece of algebra. A fairly common error was to assume that the value of R was the same as that found in part (i). Some candidates had the friction acting upwards instead of downwards.

Q.3(i) This was answered well, and the great majority of candidates obtained the given results convincingly.

Q.3(ii) Many candidates had difficulty expressing the tensions in terms of y, which should have been simply putting $x = 1.2 + y$ into the expressions used in part (i). The initial value of x (1.35) often featured in this, even though it is not relevant to finding the equation of motion. The principles behind forming the equation of motion were well understood and the correct result was often obtained. However, as this was given on the question paper, full marks could only be earned by a fully complete and accurate derivation free from any confusion over signs. The period was almost always given correctly.

Q.3(iii) Most candidates used the formula $v^2 = \omega^2(A^2 - y^2)$ to find the speed, although the values of A and y were often incorrect. Sometimes $\omega = 6.125$ was used instead of $\omega^2 = 6.125$.

Q.3(iv) Most candidates used a displacement-time equation to find a relevant time, with a few choosing instead to use velocity-time together with the speed from part (iii). Relating the time they had found to the required time when the particle was first moving upwards at C proved to be more challenging, but about half the candidates obtained the correct answer. This part was omitted altogether by about 15% of the candidates.

Q.4(i) The method for finding the centre of mass of a lamina by integration was very well understood, and most candidates obtained the correct coordinates.

Q.4(ii) Most candidates took the approach suggested in the question, considering a cone (for which standard results could be quoted) and a solid of revolution (for which the volume and centre of mass needed to be found by integration), then using the formula for the centre of mass of a composite body. This was very often completed efficiently and the given result obtained convincingly. Several candidates considered it instead as a single solid of revolution with the function defined piece-wise, often successfully. However, some continued to consider it as a lamina.

Q.4(iii) This part was omitted by about 10% of candidates. Most candidates recognised that the angle required was OAG, but the necessary trigonometry caused a lot of difficulty, with only about half the candidates obtaining the correct answer. The simplest method was $\tan^{-1}(½)+\tan^{-1}((x-1)/2)$ but many preferred to use the sine and cosine rules. A very common error was to assume that OGA was a right-angle.

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Unit level raw mark and UMS grade boundaries June 2017 series

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)										
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

				a	b	c	d	e	u
		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0